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FOR THE ISRAELI NAVY

by

Zvi Weingart

March 1991

Thesis Advisor: Alan W. McMasters

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Inventory Models for Slow-Moving Items for the Israeli Navy

by

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from the

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March 1991

ABSTRACT

This thesis examines some cost/performance models for high cost, low demand insurance items. The motivation for this research is the lack of such analytical methodology in the Israeli Navy (I.N.). The models maximize selected supply measures of effectiveness and minimize average annual holding and backordering costs. The models have the ability to rank individual items in such a way that, under a constraint of an annual provisioning budget, only those that contribute the most to the objective function are selected for stocking.

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I. INTRODUCTION

A. MOTIVATION FOR THE RESEARCH

A survey done by the inventory planning section in the Israeli Navy's (IN) Headquarters revealed that, in 1988, more than 60% of its inventory value was invested in systems and spares that experience an average demand rate of one unit or less over a three-year period. Those items, according to the I.N. regulations, were classified as slow moving items (both consumables and repairables). A careful look at the 1000 most expensive slow movers indicated that they accounted for 80% of the value of the Navy's inventories over that time period.

While most of the IN's items are managed using inventory levels which use estimated demands rates and other parameters, slow moving, expensive items are excluded from these procedures. In a few cases, levels are determined manually, but those levels are not updated on a regular basis by the Navy's item managers.

The situation is even worse when we look at the I.N.'s capability to do an initial provisioning determination of what to buy. In that case, the I.N. has to put its trust completely in vendors and contractors because no analytical tools or models exist to determine which slow movers are to be stocked. Decisions are made, not based on analytical models,

but based on a subjective evaluation by the item managers.

More than five years ago, the head of the material Logistics Support department in the I.N. Headquarters ordered the development of an analytical method that would facilitate making decisions on what quantities to buy, both for initial provisioning (new weapon systems) and for the annual replenishment review (performed by the item managers). Since no serious work has yet been done on this topic in the I.N., this thesis began the development of a replenishment model for slow movers. The first step was to determine what was available in the literature on models to manage inventories of low demand items. However, this area does not appears to be a very popular one in the operations research journals. Although most of the valuable references were from the 1960s and 1970s, they are still the most useful in that area and give some useful suggestions on modeling.

B. THESIS OBJECTIVES

There are four thesis objectives:

- To review past analyses and determine the relevant important issues concerning the problem of managing slow moving items.

- To review the deficiencies of the I.N.'s current attitude toward replenishment procedures for slow movers.

- To derive and analyze costs and supply measures of effectiveness (MOEs) models for the replenishment problem.

- To illustrate, with an example, how the suggested models work, and to apply a sensitivity analysis to the models.

C. SCOPE

The models developed in this thesis consider only expensive slow-moving consumable items. The models assume a demand of one unit at a time for each item and a demand during lead time which is Poisson distributed. The models do not allow lost sales but backorders are assumed to be allowed. The reorder point is assumed to be zero and the reorder quantity is limited to zero or one unit only. The models consider two types of objective functions, average annual costs to hold and backorder, and supply effectiveness measures. An annual procurement budget constraint will be consider for some of the models.

D. PREVIEW

Chapter II reviews the literature and discusses problems associated with managing items having of low demand rates. Chapter III reviews the I.N.'s replenishment procedure and problems in dealing with low demand items. Chapters IV and V present models to help decide on whether to stock or backorder an expensive insurance item. Chapter VI provides a numerical example showing how the proposed models work. It also provides a sensitivity analysis for the example. Chapter VII includes a summary, conclusions and recommendations.

II. BACKGROUND

A. INTRODUCTION

The purpose of this chapter is to mention some of the more interesting points and discussions found in the professional literature on the problems of provisioning and controlling slow movers.

Any inventory organization must face the problem of stocking expensive slow-moving items. However, the literature is limited in this area and many of the publications are old ones. Some of the most useful papers were written by the field investigation group of the National Coal Board in London in 1962 [Ref. 1].

B. DIFFICULTIES IN SLOW MOVER PROVISIONING

Provisioning consists of the collection of steps taken to assure adequate supply of equipment and material to an organization to support its ultimate goals in the most efficient way. Reference 2 provides an overview of the difficulties of conducting such a process when low demand spares are involved.

There are four specific difficulties:

- The inadequacy of past records in giving reliable estimators of future consumption of spares or life characteristics of a part (this is in contrast to fast movers whose consumption rates for short periods do serve as excellent estimators of future demand).

- The inflexibility of slow movers. While overstocking of fast moving spares is quickly remedied by natural

- The inflexibility of slow movers. While overstocking of fast moving spares is quickly remedied by natural consumption, such is not the case with slow moving spares. Initial overstocking can burden an organization for a long time, with high holding costs being added to an incorrect initial investment.

- The sensitivity of slow movers to variation in lead time. While fast movers can be easily adjusted to a variation in lead time, overstocking problems can occur for slow movers in cases of decreasing lead times.

- Slow-movers can cause extra costs and waiting time when found to be out-of-stock. This is in contrast to high rate demand items which experience shorter lead times, have quite a few spares and cheaper provisioning costs.

C. CLASSIFICATION OF SLOW MOVER PROBLEMS

Mitchell, in his paper [Ref. 2], gives a comprehensive classification of slow moving types and a recommended solution for each type. Figure 2.1 summarizes his classification.

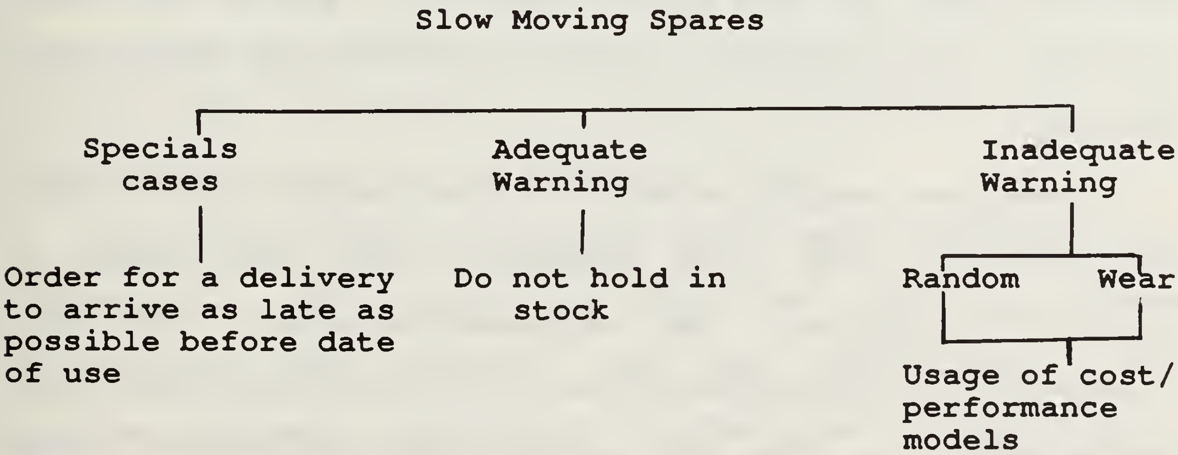

Figure 2.1 Classification of Slow Moving and Recommended Methods of Controlling Them [Ref. 2].

Special cases are those where items are bought for specific projects, overhauls or other purposes. In these cases, uncertainty is very low. The best solution is to procure the needed inventory so that delivery occurs as shortly as possible prior to use (in order to avoid holding costs).

The adequate warning cases are those in which we have warning signs of needing the item long before it is actually needed (the warning time should exceed the lead time). Here, the optimum policy is not to hold the items, but rather to order them as soon as warning signs show up (warning could be, for example, updates to the working plan for a specific fiscal year, or preliminary approval of a new operational need for the next several years). We should have a high level of certainty that the item is not needed on a day to day basis, in order to be successful with the policy of not stocking the item <u>now</u>.

Inadequate warning is both the most complex and the most interesting to deal with because of the high degree of uncertainty involved. Two subcategories exist. The first corresponds to random failure with failure rates being time-independent (i.e, the same average rate of failure holds during the entire life cycle of the spare). The second case concerns items with increasing failure rates with age (wear-out items).

In the inadequate warning cases, the best stocking solution results from optimization of an expected value objective function which may also be subject to a real limitation such as budget constraint. The usual objective functions are measures of cost or efficiency of performance.

D. DEMAND DISTRIBUTION

The Poisson distribution, as common as it is, does not always apply to every set of demand data. In the early stages of research, this author searched for other probability functions (other than the Poisson distribution) which would be adequate to describe the nature of demand for slow moving items. However, the literature contained very little in the way of suggestions for the probability mass function for those low demand rates. Some of the papers [Ref. 2] argued that, in cases where the Poisson distribution fails to give a good fit to the data, the second best option is to use the empirical distribution based on the historical data.

Two other probability mass function were suggested in the literature. The first one deals not with the demand distribution mass function, but with looking for a more precise parameter estimation of the mean of the Poisson distribution (namely λ). In some cases, when λ is not known in advance or the old data used for estimating λ has a huge variance over the mean, using the Gamma distribution can be helpful [Ref. 3]. The Gamma distribution is a two-parameter

distribution, where α is known as the shape parameter and p is the scale parameter. The Gamma distribution density function is given by:

$$f(\lambda;\alpha,p) = \frac{e^{-\lambda/\alpha}\lambda^{p-1}}{\Gamma(p)\alpha^{p}}, \qquad 0 \le \lambda < \infty, \tag{2.1}$$

where λ is the random variable and $\Gamma(p)$ is the Gamma function of p.

If p is a positive integer, then $\Gamma(p) = (p-1)!$. The expected value of λ is $E(\lambda) = p\alpha$, and the variance is $Var(\lambda) = p\alpha^2$ [Ref. 3]. Figure 2.2 illustrates the gamma density function as a function of several values of the two parameters, α and p .

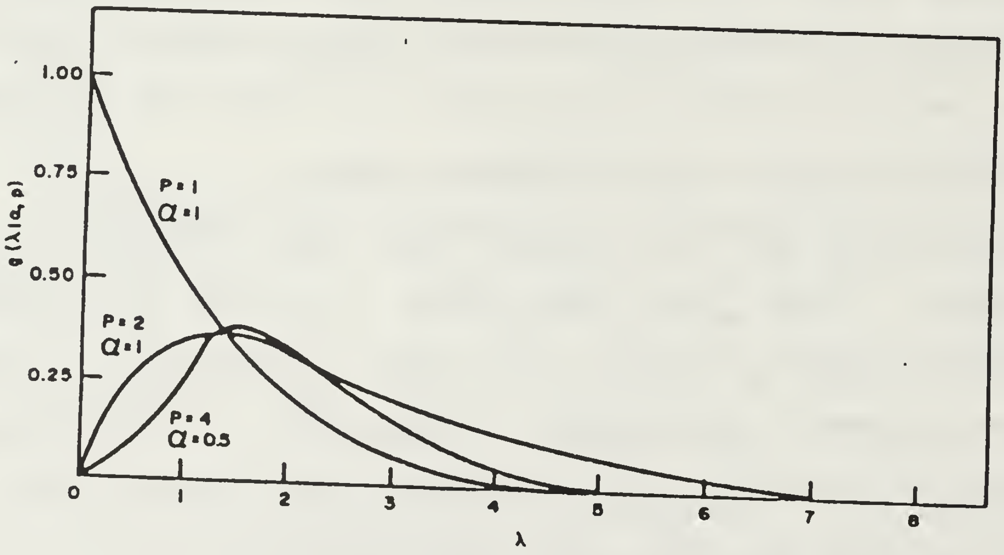

Figure 2.2. The Gamma Distribution [Ref. 3].

The advantage of using a two-parameter distribution density function is that the distribution can take many different shapes, as shown in Figure 2.2 (depends on the values of α and p), and gives more information about the data than that provided by the simple point estimator of λ. In particular, it is known [Ref. 4] that when $\alpha > 1$, it corresponds to a wear-out (increasing) failure rate type. Of course, when $\alpha = 1$, we have the exponential density function reflecting a constant failure rate over time.

The parameters of the Gamma function can be estimated by the mean μ and the standard deviation σ of the old data in the following way [Ref. 3]: :

$$\alpha = \frac{\sigma^2}{\mu} \ ; \qquad p = \frac{\mu^2}{\sigma^2} \ . \tag{2.2}$$

Another demand distribution which may apply to slow-moving items modeling is the "stuttering Poisson" distribution. While it does not fit every slow demand case, it is worth a try when demand is "lumpy" [Ref. 1]. This is also a two-parameter distribution which assumes a Poisson distribution for the number of requisitions submitted over time, and a geometric distribution for the quantity demanded in each requisition. The resulting probability mass function is:

$$R_n(t) = \frac{(1-p)\lambda t}{n} \sum_{J=1}^{n} J p^{J-1} \cdot R_{n-J} \qquad \text{for} \qquad 0 \le n \le \infty; \tag{2.3}$$

where:

$R_n(t)$ = the probability that n units are demanded in time interval t.

λ = the average arrival rate of requisitions.

p = the geometric distribution parameter describing the expected number of units demanded in each requisition.

and $R_0(t) = e^{-(\lambda t)}$.

This distribution gives better results than the Poisson distribution when the requisition arrival rate is very small but the number of units demanded in each requisition is not [Ref. 1].

E. INITIAL PROVISIONING MODELS

Burton and Jaquette's paper [Ref. 3] serves as the basis for the models developed in this thesis. It provides a procedure for deciding which items to stock in support of a new weapon system or a piece of equipment. Even though Reference 3 deals only with initial provisioning (when the items/systems are first stocked in the supply system), their models could be applicable for the annual replenishment procedure as well. These two cases can be modeled in a very similar way. In both cases they assume:

- There is a budget, limiting the amount which can be procured.

- The expected annual costs of holding and backordering the item for any stocking policy (e.g., don't stock,

  stock one, stock two, etc.) can be derived as a function of the number of units to stock.
- The supply system measures can be determined as a function of the number of the units to stock.

While the models derived later in this thesis are based on some of the models suggested by the Burton and Jaquette paper [Ref. 3], they are more cost-oriented and focus only on the case of annual stock replenishment.

F. DIFFERENT STOCKING RULES--A CASE STUDY

The literature also provided a case study which demonstrates the problems associated with stocking slow movers [Ref. 5]. In particular, the case shows how simple stock decision rules can fail for low demand items.

Data were gathered from the U.S. Navy for the "Falcon" aircraft, a large jet aircraft with more than 15,000 spare items. The data were used to generate the distribution of observed demands associated with 100 Falcons over a 13-month period. Examination of these data showed that low demand items represented about 85% of the possible candidates for a mobility package (a kind of a field repair kit for the jet). This 85% accounted for only about 10% of the quantity of items consumed, even though they were critical parts. During the 13-month period, nine out of ten "grounded" aircraft (90%) suffered shortages of low demand items. Over half of these

parts had no demand during the previous month.

Approximately 20% of the items (3,049 out of 15,000) were classified as extremely low demand items (i.e., with a demand rate of 0.035 per month or less). A provisioning policy was clearly needed for these items. Such a policy needed to consider both the very low demand rate of such items and the fact that half of the "grounded" cases were caused by these items. Three general policies were considered:

- No inventory will be stocked for these 3,049 items.
- One unit for each item will be stocked.
- Two units from each item will be stocked.

Table 2.1 shows the results from stocking according to each of these policies for the one-month protection interval desired by a kit. The probability of demand was assumed to follow the Poisson distribution with a mean of 0.035 units per month for all items. The table was computed by comparing the stocking decision (stock 0,1 or 2) with the probabilities of demanding 0,1,2 or more units over a period of one month. The numbers represent the average number of items in each category.

TABLE 2.1

SPARES PROVISIONING POLICIES FOR ITEMS WITH A 0.035 DEMAND RATE PER MONTH

Possible demand	Probability of such demand	Policy and Expected Supply Result			
		Policy	Surplus	Cons.	Shortage
0	0.965	stock zero	0	0	107
1	0.034	stock one	2942	107	3
2	0.001	stock two	3045	107	0

The supply results represent the <u>expected</u> number of items (out of 3,049) that ended up with surpluses, or shortages (the column headed "Cons" represents the expected number of items having demands equal to the quantity stocked). As Table 2.1 shows, there is an impressive reduction in the expected number of items in shortages (down from 107 items to three items when one unit of each item was stocked, and to zero when two units of each item were stocked). The negative consequences of such policies are also clear. The expected surpluses caused by no demands for the one or two units stocked of each part is extremely high. Even though dollar figures were not included in [Ref. 5], it is obvious that a large amount of money would have been invested in items that will not be consumed rapidly.

The major conclusion in this case is that these general stocking decisions do not work well when low demand rate items are involved. Stocking "one of each item" or "none of each item" created excessive surpluses. A better approach would be to base the stocking decision on an individual item basis.

This would allow for consideration of the demand distribution for each item, as well as certain relevant measures of effectiveness. The models in Chapter V are based on such an approach.

III. THE I.N. CURRENT REPLENISHMENT MODEL

As mentioned in Chapter I, the I.N. lacks analytical tools to manage its inventory of slow movers. In particular, the stocking decisions for the expensive slow movers are not justified by any economic analysis or any measure of effectiveness. The decision is made by each item's inventory manager based on his best engineering judgment and knowledge of the budget constraint for that year. Cheap slow movers are managed by automated procedures which do not differ from those used for fast movers. In many cases these automated procedures create excessive backorders or excessive on-hand inventory.

A. CURRENT REPLENISHMENT MODEL

The I.N. has several hundreds of thousands of items which are classified as active items (have at least one transaction in the last three years). Those items which do not move in three years will not be of interest in this analysis.

The active items are grouped technologically (electronics, mechanics, tools, etc.) for budgeting purposes. Annual replenishment is done in most cases by the item managers located in the Central Logistic Base (C.L.B.). Exceptions are handled by special authorized officers at Navy Headquarters.

Each item has three inventory levels. The lowest one is the "standard level" needed for operation of the fleet. This

is called the "red line" because assets should never fall below that point (similar to the idea of the "war reserves" in the U.S. Navy). The next inventory level is the reorder point. When assets fall below the reorder point, it triggers the ordering of the item. The maximum inventory level is the level an inventory manager orders up to for a given item. The I.N goal for this level is somewhere between 2.5 and 4.2 years of forecasted demand for an item (computed by demand rates from the last three years). The average goal is three years of forecasted annual demand.

The I.N. uses an old provisioning table to determine maximum inventory levels. The entry into the table requires knowing the estimated unit price of the item and the dollar value of its annual forecasted consumption rate.

The forecasted annual demand rates values used in the provisioning table are computed as follows:

$$\bar{C} = \frac{3}{15}\frac{C_3}{P_3} + \frac{5}{15}\frac{C_2}{P_2} + \frac{7}{15}\frac{C_1}{P_1} \quad , \qquad (3.1)$$

where:

C_1, C_2, C_3 = the consumption rates for the current year, last year and two years previous.

P_1, P_2, P_3 = the portion of time the item was in service in the last three years (if the item was bought for the first time in the current year, P_2 and P_3 would be zero).

The formula is a simple weighted average with the same weights being used for all items (whether they are fast or slow movers). A 0.5 round rule is used to insure C is an integer.

As items become more expensive, the maximum level dictated by the provisioning table decreases. This is intended to avoid using up the budget for only expensive items. For example, if an item costs less than 1000 Israeli shekels, the provisioning table will specify a maximum inventory level of 3.5 years of consumption. The next entry (for an item that costs more than 1000 shekels) will recommend a maximum inventory level of 2.8 years.

It should be emphasized that the computerized recommendation for inventory levels, and the quantities to purchase, can be overridden by the item manager, if he has knowledge of other constraints.

There are two problems with the provisioning table:

- Nobody knows what this table attempts to achieve. It does not support any MOE and just ensures not buying material that is too expensive.

- As indicated in the introduction (Chapter I), it does not apply for very expensive items.

Each year all active items are reviewed for replenishment according to a predetermined schedule (similar to a periodic review process). The data used in the process are the

inventory levels and a modified inventory position (defined as on-hand inventory + on-order quantities, without backorders considered). In addition, when the inventory position reaches the reorder point, the computer issues an "item review report" which alerts the item manager to consider a replenishment action.

B. DEFICIENCIES OF THE CURRENT "MODEL"

There are four major drawbacks to the current "model":

- Equation (3.1) is very insensitive to low demand. Because of the 0.5 round rule, most cases when low demand rates are involved will have a forecasted demand of zero, resulting in zero buy quantity. This may well be incorrect.

- The current model does not deal with the probabilistic nature of low demand items. The model used today is a deterministic one which is more appropriate to use with regular and stable demands rates.

- The I.N. apparently does not take into consideration any backordering or holding costs. This affects the ability to assess the desirability of stocking an item or backordering it (and supplying it later on).

- The current replenishment "model" does not consider any supply MOEs. There is no definition of what the supply system is trying to achieve when buying one quantity level instead of another. The only considerations are the judgments of the item managers and the allocated budget for that year. The annual budget is not justified in terms of different supply MOEs, but rather in terms of the previous year's budget and what people "feel" they need to stock.

The rest of this thesis investigates several possible models for "insurance" type items that do include costs and performance measures.

IV. INTRODUCTION TO COST/PERFORMANCE MODELS FOR SLOW MOVERS

This chapter contains three sections:

- Model assumptions.
- Basic steady-state formulas for on-hand inventory and backorders.
- Derivation of measures of effectiveness.

The next chapter will present the completed forms of the models.

A. MODEL ASSUMPTIONS

Before presenting the models, which are the core of this thesis, we need to state the basic assumptions needed for the derivation of the models.

1. The Replenishment Process

We are interested in a multi-item decision model for managing the inventory of slow movers. The model will assume annual continuous review model in which an annual budget will be allocated to item procurement. The quantity to buy of each item should optimize one of several different measures of effectiveness. As stated earlier, the models should also satisfy the initial provisioning situation where the slow movers are new and have not been stocked yet (they are classified as slow movers at this stage by the vendors or the contractors).

2. Constraints and MOEs

This thesis considers two kinds of problems. The first one is the unconstrained problem where each individual item is considered for stocking based only on optimizing some system objective function (annual average costs or supply MOE). The second problem considers the same objective function as the first, but also involves a provisioning budget limitation which may constrain the solution if the budget is small. The various models presented in Chapter V involve three basic ways to include backorders in the objective function:

- Expected number of backorders per year.
- Time-weighted units short per year.
- Combination of the two methods above.

The objective functions for the models represent both the economical side and the supply performance side of the stocking decision. Specifically, three objectives are considered:

- Minimize the average annual costs.
- Minimize the aggregate mean supply response time.
- Maximize the aggregate supply material availability.

3. Stocking Alternatives

Because this thesis is concerned with expensive, "insurance"-type items, the decision to be made is whether to stock one unit of an item or not to stock it at all. Stocking more than one unit is assumed to not be appropriate, even

though it could be considered as an alternative in some cases (especially if demand rates are increasing).

4. Satisfying the Demand and Provisioning Funds

Two conditions relative to funding for procuring additional units are needed for the steady state analysis. First, the assumption must be made that any demand occurring during the year will be met without exceptions. This contrasts with the case of lost sales, where unfilled demands are allowed. The second assumption states that the provisioning funds during the year will be sufficient to meet all demand. This assumption can be satisfied in two ways. Either an adequate estimated budget for the entire year is given at the beginning of the fiscal year, or an initial budget is allocated at the beginning of the year and additional funds may be requested throughout the year to meet the demand. The second option is more realistic for the I.N.

During the year, four reviews are being held. During these reviews, additional funds are supplied, if necessary, when sufficient justifications are given. In the derivation of the restricted budget problem, we will optimize the stocking levels given only the initial annual replenishment budget.

5. Poisson Demand Distribution

The Poisson distribution is the most common probability distribution used to represent demand for slow movers. This distribution is attractive due to the

exponential property of the time between demands, the fast way the function approaches zero (low times between demands account for most of the exponential distribution's density) and the assumption of the independence of events.

6. Holding Costs

Holding costs are those expenditures related directly to the item being stocked in the supply system. These costs reflect several elements, such as:

- Warehousing (space, automated equipment, forklifts, etc.).
- Administrative expenses (storekeepers, papers, computer time, etc.).
- Cost of money (interest, inflation factors, opportunity costs of not investing in other items).
- Losses due to theft, loss or misplacement.

Evaluating the holding costs of a unit has been, and will continue to be, one of the most debated topics in any cost related inventory model. Warehousing and administrative activities are hard to quantify and are therefore difficult to convert into dollars. The most common method used to account for these expenses is to assume a fraction of the unit procurement cost as a reasonable measure of the holding costs. The U.S. Navy uses 23% as the fraction for consumables items. The I.N. has never addressed the question of the proper fraction to use (given that the proper fraction might be different due to different economic condition and different warehousing expenses between the two countries). For the

purposes of this thesis, the fraction will be treated as a constant parameter and will not be addressed further.

7. Stockouts Costs

In cases where a demand has occurred and the shelf has found to be empty, an order already placed can be expedited. This procedure definitely produces additional expenses which are tough to quantify. However, they do need to be reflected by the models. It should be emphasized here that in the current I.N. supply system there has been no attempt to assess those kinds of costs and, as a consequences, they have not been reflected in any models used up to this point. The only expediting costs that can be reasonably easily quantified are those associated with "spot buys" that the Navy makes for small quantities of consumables which are bought locally in markets in cities. Unfortunately, such buys are not typically for slow moving, expensive spares.

Regardless of the difficulty associated with their evaluation, any cost model we consider has to take such backorder cost into consideration--not once, but twice. First, in the replenishment budget (which has to absorb some of this expense as a consequences of expediting) and secondly, in the backorder component of the objective function. There is a direct connection between the magnitude of any backorder costs and the optimal cost decision. The higher these costs are, the more likely that the minimum costs will be achieved by stocking the item (and thereby eliminating the need for

incurring such high costs).

B. STEADY STATE FORMULAS

In order to define what we call "costs" incurred during the year, as well as the supply measure of effectiveness, we first have to define three inventory steady state terms:

- Expected on-hand inventory at any instant of time, or D(Q,R).

- Expected number of backorders at any instant of time, or B(Q,R).

- Expected number of backorders during a year, or E(Q,R).

Reference 6 gives the exact formulas for each of these, developed under the following assumptions:

- A Poisson process generates the demands over time and each demand is for only one unit.

- Reordering is based on the value of the inventory position (IP), which is defined as on-hand inventory + on-order - backorders.

- Procurement lead time is constant and known in advance for each of the items.

- R is the reorder point. When the inventory position falls to R in value, an order in the amount of Q is placed. Thus, the minimum value of IP is R and its maximum value is (R+Q).

First, the general formulation of D(Q,R), B(Q,R), and E(Q,R) will be presented, and then they will be reduced to the special cases of R = 0 and Q = 0 or 1. We need to define the following additional terms:

D = expected demand rate per year.

PCLT = procurement lead time of the item in years (known

in advance).

J = an arbitrary integer representing the quantity of inventory position above R.

X = the number of units on hand at any point of time.

$\rho(Z)$ = the steady state probability that the inventory position is Z at any time.

$\psi_1(X)$ = the steady state probability of having X units on hand at any time.

$\psi_2(Y)$ = the steady state probability of having Y units backordered any time.

$P(W;\ D \cdot PCLT)$ = The Poisson probability that W units will be demanded during PCLT.

From the definition of J, the sum R+J represents some value of the inventory position. Since J can range from 1 to Q, R+J ranges in value from R+1 to R+Q. The fact that J is never zero is a consequence of the assumption of discrete (Poisson) demand. When demand is discrete, an order is placed at the instant that IP reaches R in value. When that happen, IP goes immediately to R+Q. Thus, the amount of time that the IP has to stay at value R is virtually zero.

To determine the expected on-hand inventory at time t, we consider the inventory position at time (t - PCLT) and a demand of (R+J-X) units during PCLT. Since all units on order at time (t - PCLT) will have arrived by time t, the net inventory (defined as on-hand - backorders) at time t will be X and if X < R+J, then it will be positive and equivalent to the on-hand inventory at time t. The probability of a net inventory being X at time t and the inventory position being

R+J at time (t - PCLT) is:

$$\rho(R+J) \cdot p(R+J-X;\ D \cdot PCLT). \tag{4.1}$$

Since J can range from 1 to Q, the probability that the on-hand inventory is X at time t, regardless of the value of IP at t - PCLT, can be determined from:

$$\psi_1(X) = \sum_{J=1}^{Q} \rho(R+J) \cdot p(R+J-X;\ D \cdot PCLT) \tag{4.2}$$

To determine $\rho(R+J)$ we need to analyze the Poisson process described by Figure 4.1.

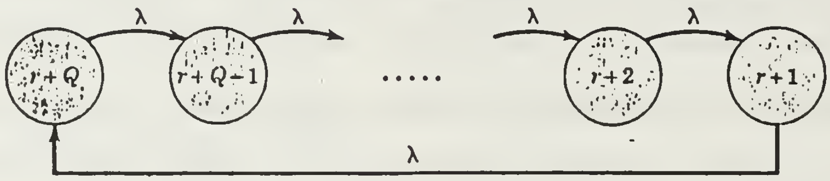

Figure 4.1 The Steady State Transition Diagram for Inventory Position [Ref. 5].

In the time dt IP can move from a given state (R+J) to the state (R+J-1) with a probability of $\lambda \cdot dt$ (where λ is the expected demand rate and demand is Poisson distributed over dt). An exception occurs when the given state is R+1. If another demand occurs, the IP value goes to R and immediately an order for Q is placed. This causes the value of IP to go

to R+Q.

Reference 6 has shown the balance equations which result are:

$$\lambda \cdot \rho(R+J+1) = \lambda \cdot \rho(R+J) \quad \text{for} \quad J = 1, \ldots, Q, \tag{4.3}$$

and

$$\lambda \cdot \rho(R+Q) = \lambda \cdot \rho(R+1). \tag{4.4}$$

Since $\sum_{J=1}^{Q} \rho(R+J) = 1$, solving these Q equations results in

$$\rho(R+J) = 1/Q \qquad \text{for} \quad J = 1, \ldots, Q; \tag{4.5}$$

which means that the probability of being in each of the possible IP states is the same and depends only on the reorder quantity Q [Ref. 6].

Substitution of $\rho(R+J) = 1/Q$ into (4.1) results in the following formula for the probability distribution for on-hand inventory:

$$\begin{aligned} \psi_1(X) &= \frac{1}{Q} \sum_{J=1}^{Q} p(R+J-X;\ D \cdot PCLT) \quad \text{for} \quad 0 \le X \le R; \\ \psi_1(X) &= \frac{1}{Q} \sum_{J=X-R}^{Q} p(R+J-X;\ D \cdot PCLT) \quad \text{for} \quad R+1 \le X \le R+Q, \end{aligned} \tag{4.6}$$

and $D(Q,R)$, the expected on-hand inventory at any time t, is then:

$$D(Q,R) = \sum_{X=0}^{R+Q} X\psi_1(X). \tag{4.7}$$

The same procedure can be used to derive $\psi_2(Y)$, the probability that Y units are backordered at any instant of time. In order to backorder Y units at time t, we need a demand of (R+J+Y) units between (t - PCLT) and t. If IP is (R+J) at time (t - PCLT), the probability of this event is:

$$\rho(R+J)\cdot p(R+J+Y;\ D\cdot PCLT) \tag{4.8}$$

Since J ranges from 1 to Q, the probability distribution $\psi_2(y)$ of Y backorders at time t regardless of the value of IP at time (t - PCLT) can be obtained from :

$$\psi_2(Y) = \sum_{J=1}^{Q} \rho(R+J)\cdot p(R+J+Y;\ D\cdot PCLT)$$

$$= \frac{1}{Q}\sum_{J=1}^{Q} p(R+J+Y;\ D\cdot PCLT) \quad \text{for } Y \geq 0. \tag{4.9}$$

We define P(out) as the steady state probability of being out of stock (no inventory on hand) at any instant of time. Then:

$$P(\text{out}) = \sum_{y=0}^{\infty} \psi_2(y) \ . \tag{4.10}$$

The average number of backorders incurred per year, E(Q,R), can be shown to be the product of the average annual demand and P(out) (shown in [Ref. 6:equation 4.29], or:

$$E(Q,R) = D \cdot P(\text{out}) \ . \tag{4.11}$$

The expected number of backorders at any instant of time B(Q,R), is defined as:

$$B(Q,R) = \sum_{Y=0}^{\infty} Y \cdot \psi_2(Y) \tag{4.12}$$

As noted in [Ref. 6], it is also the average unit years of shortage incurred per year (we will call this time-weighted units short per year in the next section).

Since we have to evaluate the holding costs, we need to evaluate the average on-hand quantity at any instant of time, D(Q,R). Equations (4.6) and (4.7) are difficult to use in the general case. However, we can develop a formula for the on-hand quantity using the relationship between the expected IP and the expected on-hand inventory.

First:

$$E(IP) = \sum_{J=1}^{Q} (R+J) \cdot \rho(R+J) = \frac{1}{Q}(R+1) + \frac{1}{Q}(R+2) + \ldots + \frac{1}{Q}(R+Q)$$

$$= \frac{Q+1}{2} + R. \qquad (4.13)$$

Since we know that:

$$D(Q,R) = E(IP) - E(\text{on-order quantity}) + B(Q,R),$$

and, since the expected on-order quantity in the steady state is equal to the expected demand during lead time [Ref. 6], the above formula for D(Q,R) can be rewritten as:

$$D(Q,R) = \frac{Q+1}{2} + R - D \cdot PCLT + B(Q,R). \qquad (4.14)$$

C. MEASURES OF EFFECTIVENESS FORMULATION

Using the formulas just derived above, we can now write the formulas for the following objective functions:

- Aggregate average mean supply response time (MSRT).
- Aggregate supply material availability (SMA).
- Average annual costs of holding and backordering.

1. Aggregate Mean Supply Response Time

MSRT has a direct linkage with another MOE, called the operational availability of a system. Reference 7 defines operational availability as the probability that a system or

equipment, when used under stated conditions in an actual operational environment, will operate satisfactorily when called upon. It is expressed as:

$$A_o = \frac{MTBM}{MTBM + MDT} \tag{4.15}$$

where:

MTBM = Mean time between maintenances. This is an engineering design characteristic or parameter of the equipment.

MDT = Mean down time (mix of administrative and engineering factors).

MDT includes the mean maintenance time (engineering design factor) it takes to repair a system. From an administrative standpoint, MDT includes the delay caused when a spare is not available on the shelf since additional time will be required to obtain the part. That is,

$$\text{MDT} = \text{mean active maintenance time} + \text{logistics and administrative delay time}\ . \tag{4.16}$$

Since we stated in the first assumption of the model that we need a multi-item decision model, typically such a model will be based on some aggregate measure of effectiveness. We therefore want an approach for converting the mean

supply response time achieved for each item into an aggregate measure of effectiveness. According to Reference 8, such an Aggregate MSRT can be defined as:

$$MSRT_{aggre} = \frac{\sum_{i=1}^{n} D_i \ MSRT_i}{\sum_{i=1}^{n} D_i} \quad \text{for } i = 1,2,\ldots,n, \tag{4.17}$$

which is a demand-weighted average of the MSRT achieved for each item (the higher the demand, the more weight is given its MSRT in the aggregate picture). MSRT is directly related to the time weighted units short (TWUS). The time-weighted units short (TWUS) is equivalent to B(Q,R) from the last section. Its unit of measure is unit—years/year. TWUS takes into account the number of units backordered as well as the time they were in backordered status. Therefore, by dividing this expression by the average annual demand rate of the item, we obtain the mean time a demand for a unit will remain backordered. We realize that this is the item's mean supply response time MSRT. These relationships are summarized by the following formulas:

$$MSRT = \quad MSRT = \frac{TWUS}{D} = \frac{B(Q,R)}{D} \ . \tag{4.18}$$

Using TWUS is appropriate in cases where each day that passes without the demanded item increases the damage due to

factors such as loss of profit, loss of operational availability, etc. The difficulty, as stated before, is to attach a dollar figure to this loss.

2. Supply Material Availability (SMA)

Supply material availability measures the extent to which the supply system can meet the demand for stocked items (without the need of backordering them). In the previous section we stated a formula for the expected number of backorders per year, E(Q,R) for an item. We use it to determine SMA as follows :

$$SMA = 100 \times \left[1 - \frac{E(Q,R)}{D}\right] . \tag{4.19}$$

We can maximize this MOE by minimizing the expected number of backorders per year. As with the MSRT, we need an aggregate SMA when we consider multiple-item models. The aggregate expression for SMA is:

$$SMA_{agg} = 100 \times \left[1 - \frac{\sum_{i=1}^{n} E_i(Q,R)}{\sum_{i=1}^{n} D_i}\right] . \tag{4.20}$$

SMA is the current major MOE used by the U.S. Navy to measure the wholesale inventory system's effectiveness.

3. Cost Formulations

We will consider two types of costs--average annual holding costs and average annual backordering costs. As

mentioned earlier, we assume that there is a need to supply all demands during the year. Therefore, the unit acquisition cost of the item will be of no interest for this analysis (the total annual procurement costs are not changed by the decision to stock or not to stock the item when we assume all demands must be met). The ordering cost usually plays a role in the decision concerning how much to buy when an order is placed. If ordering costs are very large, order size will be also large. However, since we are assuming a stocking policy where we will either have $Q = 0$ or 1, there is no opportunity for potential savings from order sizes.

a. Holding Costs

We consider the average annual holding costs as a fraction of the unit purchase price (or cost) times the average number of units on hand at any instant of time. Thus:

$$\text{Average annual holding costs} = h \cdot C \cdot D(Q,R) \; ; \qquad (4.21)$$

where:

- C = the unit cost of the item.
- h = the fraction of the unit cost which will be used to reflect the time-weighted unit holding costs.
- $D(Q,R)$ = the average expected units on hand at any instant of time.

b. Backorder Costs

These costs can be represented in three different ways. The first is:

$$TWUS_{cost} = A' \cdot B(Q,R) \tag{4.22}$$

where:

A' = the time-weighted backorder cost for one unit of the item.

The second is:

$$EBO_{cost} = A \cdot E(Q,R) \ , \tag{4.23}$$

where:

A = the backorder cost for one unit of the item (time does not play a roll here).

The third is a combination of (4.22) and (4.23); namely,

$$\text{Total Backorder Cost} = A' \cdot B(Q,R) + A \cdot E(Q,R) . \tag{4.24}$$

Equation (4.24) is applicable in many cases because it is not uncommon to let A represent the expediting ordering cost incurred and A′ represent the time-weighted damage caused to the organization because of the shortage.

D. STEADY STATE FORMULAS FOR THE SPECIFIC STOCKING POLICIES

The two alternative stocking policies we consider for expensive insurance items have a reorder point of zero and either a zero order quantity ($Q = 0$), or an order quantity of one unit ($Q = 1$). For these alternatives the steady state formulas from Section B reduce to the simple forms shown below.

From (4.9), when $J = 1$ we get:

$$\psi_2(Y)_1 = p(Y+1;\ D\cdot PCLT)\ . \tag{4.25}$$

When $J = 0$, Equation (4.5) does not hold and in this particular situation $\rho(0) = 1.0$ (since $R+Q = 0$ is the only possible value for the IP). This results in:

$$\psi_2(Y)_0 = p(Y;\ D\cdot PCLT) \tag{4.26}$$

From (4.10), (4.25) and (4.26):

$P(out)_0$ = Probability of being out of stock when we don't stock the item

$$= \sum_{y=0}^{\infty} \psi_2(y)_0 = \sum_{y=0}^{\infty} p(y;\ D\cdot PCLT) = 1 \tag{4.27}$$

$$P(out)_1 = \sum_{y=0}^{\infty} \psi_2(y)_1 = \sum_{y=0}^{\infty} P(y+1\ ;\ D\cdot PCLT) = 1 - p(0) \quad (4.28)$$

where $p(0) = e^{-D\cdot PCLT}$ for the Poisson distribution.

From (4.12), (4.25) and (4.26):

$$B(0,0) = \sum_{y=0}^{\infty} y\psi_2(y)_0 = \sum_{y=0}^{\infty} y\cdot p(y;\ D\cdot PCLT) = D\cdot PCLT \qquad (4.29)$$

$$B(0,1) = \sum_{y=0}^{\infty} y\psi_2(y)_1 = \sum_{y=0}^{\infty} y\cdot p(y+1\ ;\ D\cdot PCLT)$$

$$= p(2) + 2p(3) + 3p(4) + \ldots$$

$$= p(1) + 2p(2) + 3p(3) + 4p(4) + \ldots$$
$$- [p(1) + p(2) + p(3) + p(4) + \ldots]$$
$$= \sum_{y=0}^{\infty} y\cdot p(y;D\cdot PCLT) - [1-p(0)]$$
$$= D\cdot PCLT - [1-p(0)] \qquad (4.30)$$

From (4.11), (4.27) and (4.28) :

$$E(0,0) = D\cdot P(out)_0 = D\cdot 1 = D\ ; \qquad (4.31)$$

$$E(0,1) = D\cdot P(out)_1 = D\cdot [1-p(0)] \qquad (4.32)$$

$$D(0,0) = 0, \quad \text{since no inventory is being held,} \qquad (4.33)$$

And from (4.14), (4.30) and (4.32):

$$\begin{aligned} D(0,1) &= \frac{Q+1}{2} + R - D\cdot PCLT + B(0,1) \\ &= 1 + 0 - D\cdot PCLT + \{D\cdot PCLT - [1-p(0)]\} \\ &= p(0). \end{aligned} \qquad (4.34)$$

E. BEHAVIOR OF COSTS/SUPPLY MOES

Using the formulas from the previous section we can state the reduced forms of the different costs and supply performance measures (MOE) for the two proposed stocking policies. We will also examine their behavior as annual demand or demand during lead time varies.

1. Annual Expected Holding Costs

$$\text{Average annual holding costs } (0,0) = 0\ ; \qquad (4.35)$$

$$\begin{aligned} \text{Average annual holding costs } (0,1) &= h\cdot C\cdot D(0,1) \\ &= h\cdot C\cdot e^{-D\cdot PCLT}. \end{aligned} \qquad (4.36)$$

Figure 4.2 illustrates the differences in the average number of units on hand, and therefore the annual expected

holding costs as a function of D when $h \cdot C = 1$.

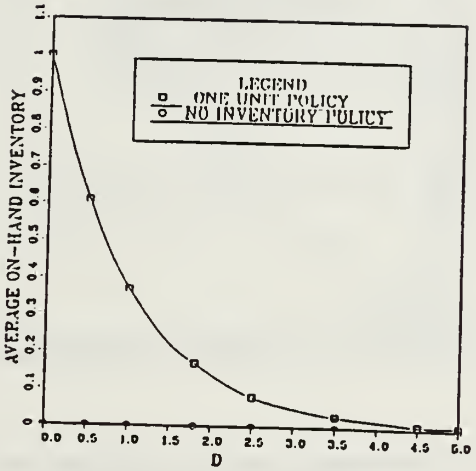

Figure 4.2 Average On-Hand Inventory as a Function of D (PCLT = 2).

2. Annual Expected Backorder Costs

$$EBO_{cost}(0,0) = \; = A \cdot E(0,0) = A \cdot D \; ; \tag{4.37}$$

$$EBO_{cost}(0,1) = A \cdot E(0,1) = A \cdot D \; [1-p(0)] \; . \tag{4.38}$$

Figure 4.3 illustrates the differences in the expected number of backorders incurred annually between the two stocking policies as a function of D. This graph also represents the differences in costs between the two stocking policies when A = 1.

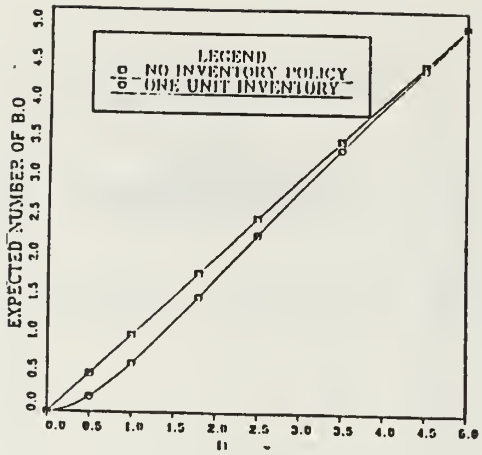

Figure 4.3 Average Annual Expected Backorders as a Function of D (PCLT=2).

3. Expected Annual Time Weighted Units Short Costs

$$TWUS_{cost}\ (0,0) = A' \cdot B(0,0) = A' \cdot D \cdot PCLT \ ; \qquad (4.39)$$

$$\begin{aligned} TWUS_{cost}\ (0,1) &= A \cdot B(0,1) \\ &= A' \cdot [D \cdot PCLT - 1 + p(0)]. \qquad (4.40) \end{aligned}$$

Figure 4.4 illustrates the differences in the annual expected TWUS between the policies as a function of D (PCLT = 2).

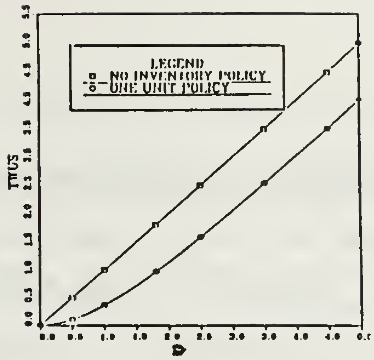

Figure 4.4 Average Annual TWUS as a Function of D (PCLT = 2).

The figure also reflects the differences in the TWUS costs for A′ = 1.

4. <u>Mean Supply Response Time</u>

From previous derivations we have seen that

$$\mathrm{MSRT}(0,0) = \frac{B(0,0)}{D} = \frac{D \times \mathrm{PCLT}}{D} = \mathrm{PCLT} \; ; \tag{4.41}$$

$$\mathrm{MSRT}(0,1) = \frac{B(0,1)}{D} = \frac{D \cdot \mathrm{PCLT} - [1-p(0)]}{D} \; . \tag{4.42}$$

In order to graph the differences between the two formulas, we first have determine the limits of MSRT(0,1) and MRST(0,0) as D approaches zero.

$$\lim_{D \to 0} \mathrm{MSRT}(0,1) = \lim_{D \to 0} \frac{D \cdot \mathrm{PCLT} - 1 + e^{-D \cdot \mathrm{PCLT}}}{D} = \mathrm{PCLT} - \lim_{D \to 0} \frac{1 - e^{-D \cdot \mathrm{PCLT}}}{D}$$

$$= \mathrm{PCLT} - \mathrm{PCLT} = 0 \; .$$

Also,

$$\lim_{D \to 0} \mathrm{MSRT}(0,0) = 0 \; .$$

This special result is true because if there is no demand we do not have any problem satisfying it instantly, even if we do not have an item on hand. In the plot of MSRT(0,0), when the demand rate is above 0, MSRT will be equal to PCLT. MSRT(0,1) will start at zero when no demand is present, and will increase as the demand rate increases (as shown by Figure 4.5).

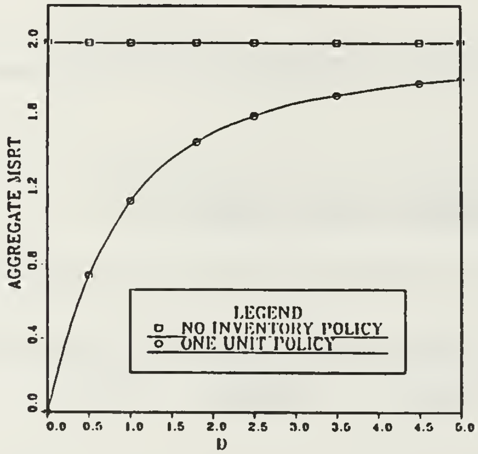

Figure 4.5 MRST as a Function of D (PCLT = 2).

V. COST/PERFORMANCE STOCKING MODELS

Two types of models are derived in this chapter. The first one is unconstrained. The second type includes a budget constraint.

The unconstrained models are only concerned with minimizing average annual costs. Models which incorporate a budget constraint are concerned with minimizing average annual costs and optimizing supply MOEs as well. This chapter will also discuss the optimal solution procedure for all model types.

A. UNCONSTRAINED MODELS

The unconstrained models for deciding whether or not to stock an item are based on an economic analysis of the options. Clearly, the average annual costs incurred during a year are influenced by the stocking decision.

The general form of such models is:

Find Q_i, i = 1, 2, ..., n which minimize

$$\sum_{i=1}^{n} \text{cost}_i(Q_i) \; ; \tag{5.1}$$

where:

Q_i = the inventory order quantity of item i,

n = the number of items we are considering,

$\text{cost}_i(Q_i)$ = the total average annual costs incurred when Q_i is stocked for item i.

While we are mainly interested in a model which deals with several spares (of a specific weapon system or mix of items), it should be obvious that the models can also be used to evaluate the desirability of stocking individual items.

The sum of annual average costs for all items, as presented by (5.1), can be minimized by minimizing each one of the individual costs since, as we will see shortly, the cost terms contain no cross-products (i.e, $Q_i . Q_j$). This property, known as separability, allows us to write (5.1) as:

$$\min \sum_{i=1}^{n} \text{cost}_i(Q_i) = \sum_{i=1}^{n} \min \text{cost}_i(Q_i) \quad . \tag{5.2}$$

We will therefore concentrate on minimizing $\text{cost}_i(Q_i)$, and seek the optimum decision for the cost function. Since Q_i can take on only values of 0 and 1, we can reduce our problem to three marginal cases where we will compare the costs when $Q_i = 0$ with the costs when $Q_i = 1$. We will also suppress the subscript i for the rest of the presentation of the unconstrained models. The three cases reduce to:

$$\begin{aligned} &(a) \quad \text{cost}(0) - \text{cost}(1) < 0 \ , \\ &(b) \quad \text{cost}(0) - \text{cost}(1) = 0 \ , \\ &(c) \quad \text{cost}(0) - \text{cost}(1) > 0 \ . \end{aligned} \tag{5.3}$$

In the first case, the "no holding" (or Q = 0) policy is cheaper than the stocking policy (or Q = 1), and therefore

$Q = 0$ is optimal (i.e., all demands will be backordered for the item). In the second case, there is no difference between the costs of the two alternatives, so both $Q = 0$ and $Q = 1$ are optimal. We will be able to draw the corresponding indifference curve later. In the third case, it is cheaper to stock the item ($Q = 1$) than not ($Q = 0$).

Three specific cost models will be examined next. The first considers backorders, but does not consider the length of time any demands remain unfilled. The second considers time-weighted backorders and the third considers both of the above.

1. Expected Backorder Costs Case (EBO)

If we are not concerned with the dimension of time, we reflect the backorder costs by the expected number of backorders per year. In this case the formulas derived in Chapter IV give:

$$
\begin{aligned}
&\text{(a)} \quad \text{cost}(0) = \text{backordering cost} = A \cdot D \\
&\text{(b)} \quad \text{cost}(1) = \text{holding + backordering costs} \\
&\qquad\qquad = C \cdot h \cdot p(0) + A \cdot D \cdot [1 - p(0)]
\end{aligned}
\tag{5.4}
$$

Setting (5.4a) equal to (5.4b) and solving for D, we get:

$$A \cdot D = C \cdot h \cdot p(0) + A \cdot D \cdot [1-p(0)] ,$$

which reduces to :

$$[C \cdot h - A \cdot D] \cdot p(0) = 0 .$$

Since $p(0) > 0$, cost (1) equals cost (0) only if

$$C \cdot h = A \cdot D .$$

This can be rewritten as :

$$D = \frac{Ch}{A} . \tag{5.5}$$

Thus, this value of D is the breakeven point between the costs of the first and the third policies from (5.3). In other words, when the rate of annual demand equals the critical value shown in (5.5), we do not differentiate between the two policies. They are equally attractive.

Considering (5.5) with (5.3) allows us immediately to state the optimal decision for the complete spectrum of the demand rate. It is shown in Table 5.1.

TABLE 5.1

OPTIMAL STOCKING DECISIONS FOR ALL POSSIBLE DEMAND RATES FOR THE EBO MODEL

Case	Demand Rate	Optimal Decision	Reason
a	$D < \frac{Ch}{A}$	Don't stock. (Q = 0)	Backordering is cheaper than holding the unit.
b	$D = \frac{Ch}{A}$	Either policy is optimal.	The two expected costs are equal.
c	$D > \frac{Ch}{A}$	stock. (Q = 1)	Holding one unit is cheaper.

Figure 5.1 illustrates the optimal decision rule for a range of A, D and two different unit costs values. The value of h for this illustration is 0.23.

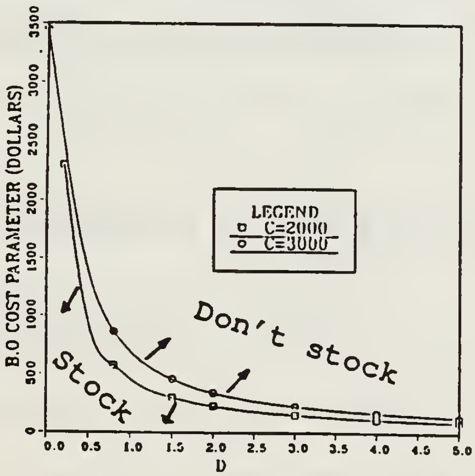

Figure 5.1. Optimal stocking Policy for Different Values of A, D and C for the EBO Model.

As is expected, as the unit price increases, the optimal decision would change toward not holding the unit. For a fixed unit cost and demand rate, as the backorder cost A increases, stocking one unit becomes more desirable.

2. <u>Expected Time-Weighted Units Short (TWUS) Cost Model</u>

In this case, the backorder costs include the effect of time that they remained unfilled. From Chapter IV, the objective function was:

$$\begin{aligned}
&\text{(a)} \quad cost(0) = A' \cdot D \cdot PCLT, \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (5.6) \\
&\text{(b)} \quad cost(1) = C \cdot h \cdot p(0) + A' \cdot [D \cdot PCLT - 1 + p(0)].
\end{aligned}$$

The same cases from (5.3) apply here also. Case (b) gives Q = 0 or Q = 1 as optimal stocking decision. When this happens:

$$A' \cdot D \cdot PCLT = C \cdot h \cdot p(0) + A' \cdot [D \cdot PCLT - 1 + p(0)] \ .$$

This reduces to

$$p(0) \cdot [h \cdot C + A'] - A' = 0 \ .$$

or

$$e^{-D \cdot PCLT} = \frac{A'}{Ch + A'}$$

Taking the natural logarithm of both sides gives

$$D \cdot PCLT = -\ln \left[\frac{A'}{Ch + A'}\right] \qquad (5.7)$$

Again, this value of D·PCLT is the breakeven point when the expected costs of the two policies (Q = 0 and Q = 1) are equal. The optimal decision in this case is represented for all possible values of D·PCLT by Table 5.2.

TABLE 5.2

OPTIMAL STOCKING DECISIONS FOR ALL POSSIBLE (D·PCLT) VALUES FOR THE TWUS MODEL

Case	Demand Rate	Optimal Decision	Reason
a	$D \cdot PCLT < -\ln \left[\frac{A'}{Ch + A'}\right]$	Don't stock. (Q = 0)	Backordering is cheaper.
b	$D \cdot PCLT = -\ln \left[\frac{A'}{Ch + A'}\right]$	Either policy is optimal.	Both costs are equal.
c	$D \cdot PCLT > -\ln \left[\frac{A'}{Ch + A'}\right]$	Stock. (Q = 1)	Holding one unit is cheaper.

Figure 5.2 illustrates the decision rules for different values of A′, D and PCLT for two different unit cost values. Again, h is 0.23 in this illustration.

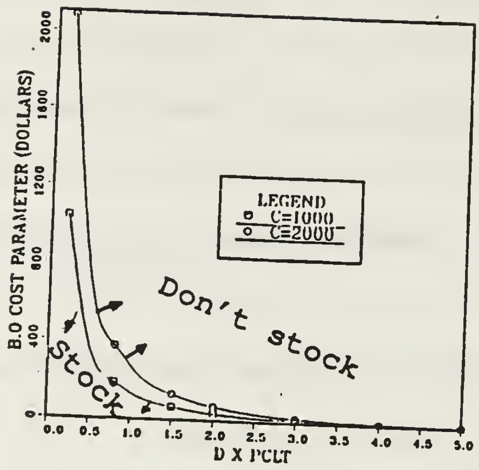

Figure 5.2. Optimal Stocking Policy for Different Values of A, D, PCLT and C for the TWUS Model.

Even though the basic behavior is similar in the two cases, there is a definite difference in the way the models behave as a function of shortage cost, D and PCLT. If the backorder cost is time-dependent, then the decision not to stock occurs sooner than when time is not part of the backorder cost.

3. EBO and TWUS Model

If we want to consider both the fixed cost of having a backorder (represented by A) and the time-dependent component of that cost (represented by A′), we have to consider the following cost formulation:

$$\text{a)}\quad cost(0) = A'\cdot D\cdot PCLT + A\cdot D = D\cdot(A'\cdot PCLT + A)\ ,$$

$$\tag{5.8}$$

$$\text{b)}\quad cost(1) = C\cdot h\cdot p(0) + A'\cdot[D\cdot PCLT - 1 + p(0)] + A\cdot D\cdot[1-p(0)]\ .$$

Setting (5.8a) equal to (5.8b) will give us:

$$C\cdot h\cdot p(0) - A' + A'\cdot p(0) - A\cdot D\cdot p(0) = 0,$$

or

$$D = \frac{p(0)\left[Ch + A'\right] - A'}{A\cdot p(0)}\ , \tag{5.9}$$

where $p(0) = e^{-D\cdot PCLT}$ for the Poisson demand case.

Table 5.3 gives the optimal decisions in this case as a function of D.

TABLE 5.3

OPTIMAL STOCKING DECISIONS FOR ALL POSSIBLE D VALUES. FOR THE EBO + TWUS MODEL

Case	Demand Rate	Optimal Decision	Reason
a	$D < \dfrac{p(0)\left[Ch + A'\right]}{A \cdot p(0)}$	Don't stock (Q = 0).	Backordering is cheaper.
b	$D = \dfrac{p(0)\left[Ch + A'\right] - A'}{A \cdot p(0)}$	Either policy is optimal.	Both costs are equal.
c	$D > \dfrac{p(0)\left[Ch + A'\right] - A'}{A \cdot p(0)}$	Stock. (Q = 1)	Holding one unit is cheaper.

Figure 5.3 illustrates the decision rules for different values of D and C, for two different sets of backorder costs (A and A′). PCLT for this illustration is set to two years and h is again 0.23.

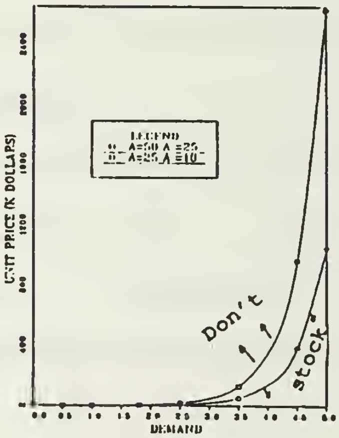

Figure 5.3. Optimal Stocking Policy for Different Values of D, C, A and A′ in the EBO+TWUS Model

The three optimal policy guides, given in Tables 5.1, 5.2 and 5.3, are very powerful and easy to use. Reference 9 provides an example of the advantages provided by Table 5.1. It is a case study done on a sample of slow movers examined at one of the largest shipyards in the world in Lisbon ("Marqueira"). The case study compared the average annual costs of two policies:

- Previous policy--stocking one or more units for each slow mover considered to be an important spare to the shipyard.

- Recommended stock quantity given by the EBO model presented above.

Each item in the sample had a value for A which was composed of the relative importance of the spare to the machine it belonged to and the relative importance of the machine to the activity of the shipyard (A ranged from $1200 to $7000). Each item in the sample cost more than $400 and h was selected to be 0.3. PCLT was assumed to be six months for all items.

For the random sample chosen, it was shown that by applying the economic model, approximately 50% of average annual costs could be saved during the year (both holding and backorder costs). While the previous stocking rule had an average annual cost of $22,300 the economic stocking model had average costs of only $11,000.[1]

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[1]The interested reader is referred to Reference 9 to get more details on the analysis and parameters used in this study.

B. THE CONSTRAINT PROBLEM

In many cases, both the holding/backordering costs projected during the year, as well as the initial replenishment budget affect the decision whether or not to stock an expensive spare. A limited replenishment budget might change the character of the final solution by forcing less procurement then would have been done without the constraint. This section provides five different models that consider both the average annual costs and supply MOEs when there is a budget constraint.

1. Constrained Costs Problem

a. Framework for the Constrained Costs Problem

In the case of the constrained problem, we are concerned with finding Q_i , i = 1,2 .... n, which

$$\begin{aligned} &\text{minimize} \quad \sum_{i=1}^{n} \text{cost}_i(Q_i) \\ &\text{subject to} \quad \sum_{i=1}^{n} C_i \cdot Q_i \le B, \end{aligned} \tag{5.10}$$

where B = Annual initial replenishment budget.

The constrained problem in (5.10), when the constraint is binding, can be viewed as an unconstrained one by considering the Lagrangian for the problem The following Lagrangian results:

$$L = \sum_{i=1}^{n} cost_i(Q_i) + \lambda \cdot [\sum_{i=1}^{n} C_i \cdot Q_i - B] \tag{5.11}$$

where λ is the Lagrange multiplier.

This function is separable and therefore we need only to take a look at the part of the function which involves item i. As Reference 3 indicates, in this type of problem, the same λ and Q_i $(i = 1,2,....,n)$ will minimize the reduced Lagrange function:

$$L_i(Q_i) = cost_i(Q_i) + \lambda \cdot C_i \cdot Q_i. \tag{5.12}$$

Following the approach for the unconstrained problem, we have three possible cases:

(a) if $L_i(1) - L_i(0) < 0$, then $Q_i = 1$ is the optimal solution,

(b) If $L_i(1) - L_i(0) = 0$, both $Q_i = 1$ and $Q_i = 0$ are optimal, (5.13)

(c) If $L_i(1) - L_i(0) > 0$, then $Q_i=0$ is the optimal solution.

From (5.12) we get that:

$$L_i(0) = cost_i(0) \ ; \quad \text{and}$$

$$L_i(1) = cost_i(1) + \lambda \cdot C_i \ .$$

Substituting into the first case of (5.13) yields:

$$\text{cost}_i(1) + \lambda \cdot C_i - \text{cost}_i(0) < 0$$

or:

$$\lambda < \frac{\text{cost}_i(0) - \text{cost}_i(1)}{C_i} \tag{5.14}$$

For the third case we get:

$$\lambda > \frac{\text{cost}_i(0) - \text{cost}_i(1)}{C_i} \tag{5.15}$$

The second case gives equality for both (5.14) and (5.15). Combining these results, we get:

$$\left[\frac{\text{cost}_i(0) - \text{cost}_i(1)}{C_i} \begin{array}{l} \text{all items } i \\ \text{for which} \\ Q=0 \text{ is} \\ \text{optimal} \end{array} \right] < \lambda < \left[\frac{\text{cost}_i(0) - \text{cost}_i(1)}{C_i} \begin{array}{l} \text{all items } i \\ \text{for which} \\ Q=1 \text{ is} \\ \text{optimal} \end{array} \right] \tag{5.16}$$

To determine the value of λ satisfying (5.16) for all items, we first have to calculate the marginal cost ratio, MCR_i, for each item i, where:

$$MCR_i = \frac{\text{cost}_i(0) - \text{cost}_i(1)}{C_i} \quad , \quad \text{for } i = 1,2,\ldots.n \tag{5.17}$$

and rank those with positive values from the largest to smallest. We first buy one unit (i.e., $Q_i = 1$) for the item with the largest MCR_i, then we buy one unit for the next largest MCR_i item, and so on until the budget is used up or no more units can be bought with the remaining budget. The value of optimal λ is the smallest positive MCR_i for which $Q_i = 1$ was feasible. Actually, for the problem in this section, we are not interested in the value of λ, but rather in the procedure for determining the items for which $Q_i = 1$ and $Q_i = 0$. The ranking procedure just described provides that procedure.

If the Q_i were not required to be integer and if the objective function of (5.10) was a continuous function of the Q_i's, then λ would give us the <u>shadow price</u> associated with the annual cost reduction provided when the budget (B) increases by one dollar. Unfortunately, in our discrete problem, it does not correspond to such a shadow price, unless:

$$\sum_{i=1}^{n} C_i \cdot Q_i = B.$$

at optimality. Exact equality is not expected when the value of B is specified before the budget allocation takes place.

Up to this point we have assumed the budget constraint is binding. However, in general we may not know if that will be true. Only after we follow the procedure described above, we might know exactly if the constraint is binding or not. What does it mean if we buy $Q_i = 1$ for all positive MCR_i and

still have not used up the budget? It means that the problem is an unconstrained one because $\lambda = 0$ will satisfy (5.11). It should be obvious that we would never buy $Q_i = 1$ for any item having a negative MCR_i, because the numerator of the ratio causes the ratio to be negative; that is:

$$cost_i(0) < cost_i(1),$$

and therefore buying nothing gives a lower value of the objective function (annual holding and backordering costs) than buying $Q_i = 1$ for such items.

Table 5.4 summarizes the results of this section. The results of this section apply to the three costs constrained models discussed in the following sections of this chapter.

TABLE 5.4
BASIC RECOMMENDED STOCKING POLICY FOR
THE CONSTRAINED PROBLEM

Case	Condition	Value of MCR_i	Recommended Policy
a	$L_i(1) - L_i(0) < 0$	Positive	* Stock the item using the ranking procedure.
b	$L_i(1) - L_i(0) = 0$	Zero	* Both policies are optimal.
c	$L_i(1) - L_i(0) > 0$	Negative	Do not stock the item.

* If there is sufficient budget to do so.

b. EBO Model

When the expected number of backorders per year is used as the backorder term in the average annual costs, (5.4) expresses the cost for each of the stocking policies. Replacing them in (5.17), and suppressing the subscript i, gives:

$$MCR_{EBO} = \frac{D \cdot A - p(0) \cdot h \cdot C - A \cdot D[1 - p(0)]}{C} ,$$

which reduces to:

$$MCR_{EBO} = \frac{e^{-D \cdot PCLT}(A \cdot D - h \cdot C)}{C} . \tag{5.18}$$

According to Table 5.4, if this ratio is positive, we will consider the item for stocking based on the ranking

procedure and the size of our budget. If this ratio is negative, we definitely will not stock it. If the ratio is zero, either stocking ($Q_i = 1$) or backordering the item ($Q_i = 0$) are optimal. Figure 5.4 illustrates the behavior of this ratio as a function of D and the backorder cost parameter A (PCLT was assumed to be two years for the sake of the illustration).

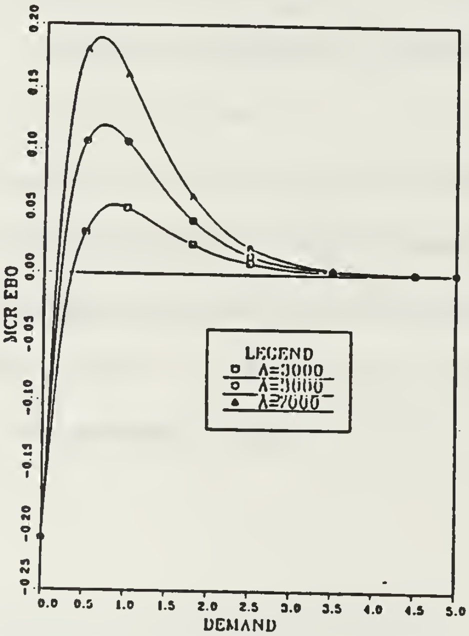

Figure 5.4. MCR_{EBO} as a Function of D for Different Values of A (C = 5000, PCLT = 2 and h = 0.23).

Figure 5.4 shows that the ratio reaches its maximum value when:

$$D = \frac{PCLT \cdot h \cdot C + A}{A \cdot PCLT} \; .$$

This formula was derived by taking the partial derivative with respect to D and setting it equal to 0. As A gets larger and larger, the maximum value of MCR is reached when D = 1/PCLT, because:

$$\lim_{A \to \infty} \frac{PCLT \cdot h \cdot C + A}{A \cdot PCLT} = \frac{1}{PCLT}$$

Figure 5.4 also illustrates the following characteristics of MCR_{EBO}:

- As demand increases,the desirability of holding the item increases. This is true until D is set equal to 1/PCLT (when maximum ratio value is achieved).
- When D > 1/PCLT, MCR_{EBO} decreases and reaches an asymptotic limit of zero (as demand increases to infinity).
- As backorder costs increase, the desirability of stocking the item also increases.

c. TWUS Model

When the backorders costs include the time a backorder remains unfilled, the average annual costs have TWUS in the backorder term. Formula (5.6) presented the costs incurred for each of our two policies. Substituting them into the general MCR equation (5.17) yields:

$$MCR_{TWUS} = \frac{A' \cdot D \cdot PCLT - h \cdot C \cdot p(0) - A'[D \cdot PCLT - 1 + p(0)]}{C}$$

which reduces to:

$$MCR_{TWUS} = \quad . \tag{5.19}$$

Figure 5.5 illustrates the behavior of this ratio as a function of D and several values of the backorder cost parameter A′.

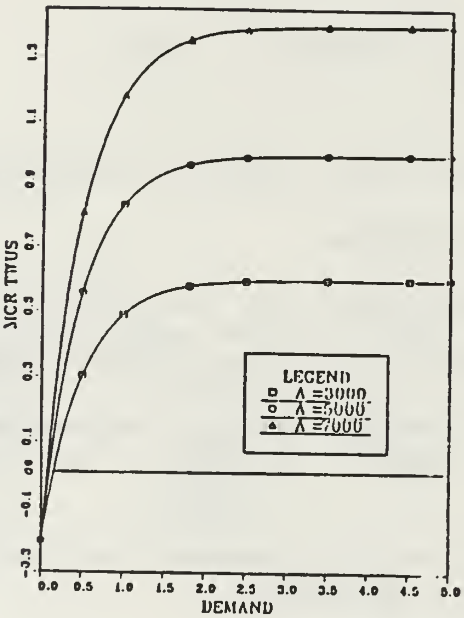

Figure 5.5. MCR_{TWUS} as a Function of Demand for Different Values of A′ (PCLT = 2, C = \$5000 and h = 0.23).

Figure 5.5 illustrates the following characteristics of MCR_{TWUS}:

- As demand during PCLT increases, the desirability of holding the item increases but at a decreasing rate.
- As The time-weighted backorder cost increased, the desirability of stocking the item also increases.
- The asymptotic limit of the ratio is (A′/C). No further

savings can be achieved in the backorder costs, when one unit is stocked. The holding cost reaches zero in the limit under the Q = 1 stocking policy. The net result is that when D become very large, the difference in the total annual costs of the two policies reaches a constant value.

d. EBO and TWUS Model

As mentioned earlier, there are situations in which we would like to assess both a penalty for each backordered unit and a time weighted penalty (the penalty per unit might represent the extra expediting costs required). It is then proper to use both the backordering cost parameters of the items. The expected annual costs when both backorder costs are included were presented in (5.8).

Replacing (5.8) in (5.17) yields the next MCR.

$$MCR_{EBO+TWUS} = \frac{e^{-D \cdot PCLT}(A \cdot D - A' - h \cdot C) + A'}{C} \tag{5.21}$$

Figure 5.6 illustrates the behavior of this ratio as a function of D (assuming PCLT is constant) for several values of the backorder cost parameters A and A'.

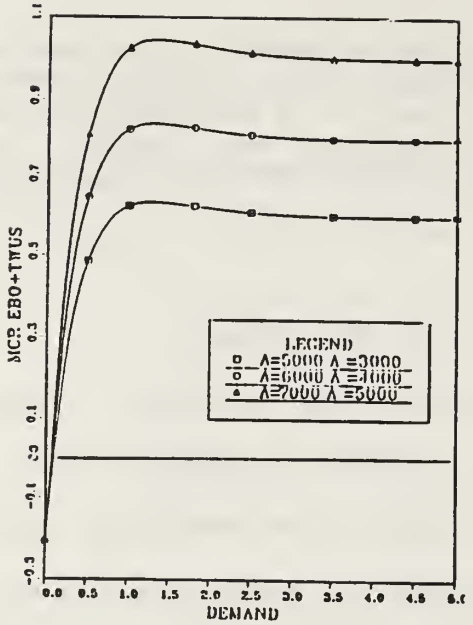

Figure 5.6. $MCR_{EBO+TWUS}$ as a Function of Demand for Different Combinations of Values for A and A′ (PCLT = 2, C = \$5000 and h = 0.23).

It is easy to notice that this MCR combines the attributes of the previous two versions discussed earlier. After reaching a maximum value (the same value of D as shown in figure 5.4), the marginal ratio decreases and remains at a fixed level no matter how much demand increases.

2. Supply MOE Models

a. Framework for the Constrained supply MOE Problem

We now develop models having supply MOEs as their objective functions. Here we solve the following problem:

Find Q_i, i = 1, 2,...,n which :

$$\begin{array}{l} \text{maximize} \\ \quad \text{or} \\ \text{minimize} \end{array} \quad \sum_{i=1}^{n} MOE_i(Q_i) \; ,$$

$$\text{subject to:} \quad \sum_{i=1}^{n} C_i \cdot Q_i \leq B.$$

The objective function is also separable in this case so we can concentrate on optimizing each item's MOE separately. The Lagrangian can again be formed with $MOE_i(Q_i)$ in place of $cost_i(Q_i)$ in Equation (5.11).

Only one possible case exists now. This is because of the monotone property of each MOE (each decision to stock an item will improve the objective function for the MOE's we are considering, in contrast to the cost objective function). So we only consider cases where:

$$L_i(1) - L_i(0) < 0,$$

For this case, $Q_i = 1$ is the optimal solution.

In the same manner as above, we also obtain the Marginal Ratio, which we now call the Marginal Performance Ratio (MPR).

$$MPR = \frac{MOE(0) - MOE(1)}{C} \; . \tag{5.22}$$

Since the MPR will be positive for all items, they are all candidates for stockage. The ranking procedure is again used, because the higher the positive MPR is, the more desirable it is to stock the item (just as in the cost models). We start spending the budget on the item having the largest MPR, than we buy one unit of the item having the next largest MPR, and so on. If we can we try to buy $Q_i = 1$ for all i. If B is large enough to allow that, then the problem is unconstrained.

b. SMA Model

From (4.37) and (4.38), we showed that:

$$EBO(0) = D \ ,$$

and

$$EBO(1) = D \cdot [1 - p(0)] \ .$$

The desired objective function is to maximize SMA. However, since maximizing SMA means minimizing the expected annual number of stockouts, we can write MPR in terms of EBO.

$$MPR_{SMA} = \frac{EBO(0) - EBO(1)}{C} = \frac{D - D[1 - p(0)]}{C} = \frac{D \cdot e^{-D \cdot PCLT}}{C} \ . \qquad (5.23)$$

Figure 5.7 illustrates the behavior of the ratio as a function of D when PCLT = 2 for several values of the unit costs (C).

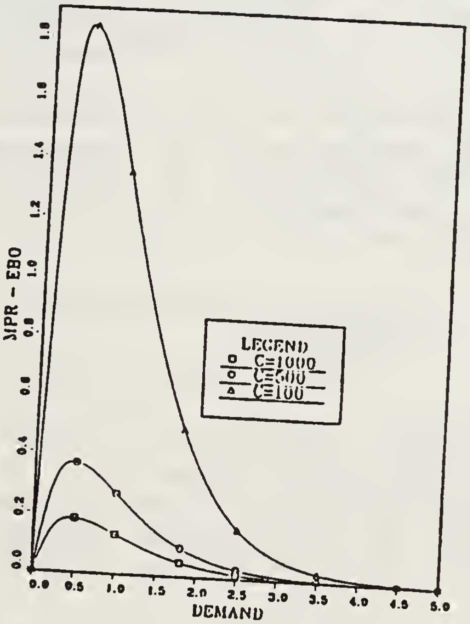

Figure 5.7 MPR_{SMA} as a Function of D (PCLT = 2) and the Unit Price.

This ratio reaches its maximum value when D equals the reciprocal 2of PCLT (D·PCLT = 1). After this point, it declines to zero. With a high rate of demand, there is a decreasing benefit in stocking only one unit of the item. That is why the ratio decreases to zero.

c. MSRT Model

As shown in Equations (4.41) and (4.42),

$$MSRT(0) = PCLT \ ;$$

$$MSRT(1) = \frac{D \cdot PCLT - [1 - p(0)]}{D} \ .$$

When we use MSRT(0) and MSRT(1) to replace MOE(0) and MOE(1) in Equation (5.22), we get:

$$MPR_{MSRT} = \frac{1 - e^{-D \cdot PCLT}}{D \cdot C} \ . \tag{5.24}$$

Figure 5.8 illustrates the behavior of this ratio as a function of D (PCLT = 2) for several values of unit costs (C).

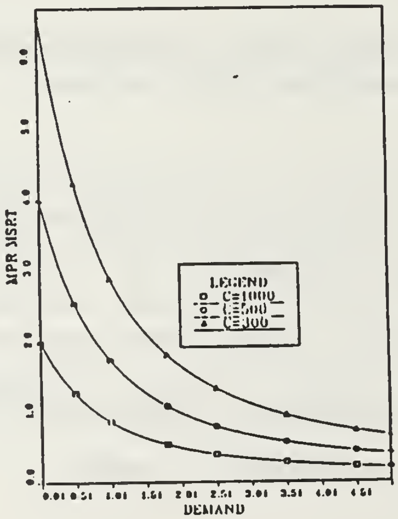

Figure 5.8. MPR_{MSRT} as Function of D (PCLT = 2) and C.

The figure shows that the ratio approaches the finite limit of (1/D·C) because no further improvements can be achieved by deciding to stock only one unit of the item.

C. SOLUTION PROCEDURE

This section summarizes the procedures needed to obtain an optimum selection of items for stocking. Figure 5.9 gives a comprehensive flow diagram to use in conducting the analysis.

As we mentioned earlier, in the supply MOE's unconstrained case, we don't need to rank the items. We can buy all of them since for each one of them $MOE_i(1) > MOE_i(0)$ and the objective function is improved by stocking them.

In the supply MOE's constrained problem, the ranking procedure must be used, since the higher the MPR is, the more desirable it is to stock the item. We start spending the budget on the item having the largest MPR and we buy one unit of it. If we can we then buy the item with the next largest MPR, and so on. We continue until we are left with no more budget to buy the next candidate for stocking.

In the budget constrained problem, a global optimum may be different from the local constrained solution we get from the ratios and the ranking procedure. We have found examples when the two solutions are different.

If we can not stock all items having a positive MCR, it indicates that the solution is a constrained one because if the constraint were relaxed, we could stock all these items

and lower the total annual average costs.

In the constrained case, the ranking procedure based on the marginal ratios recommends stocking items with the highest ratios. We need to emphasize that if the next selected item's unit cost exceeds the remaining annual budget, we do not stop, but continue to try to stock the next highest ranked item, even if it is less desirable. This is done until we consume the entire budget or, in the case of cost minimization, no more items with a positive MCR remain.

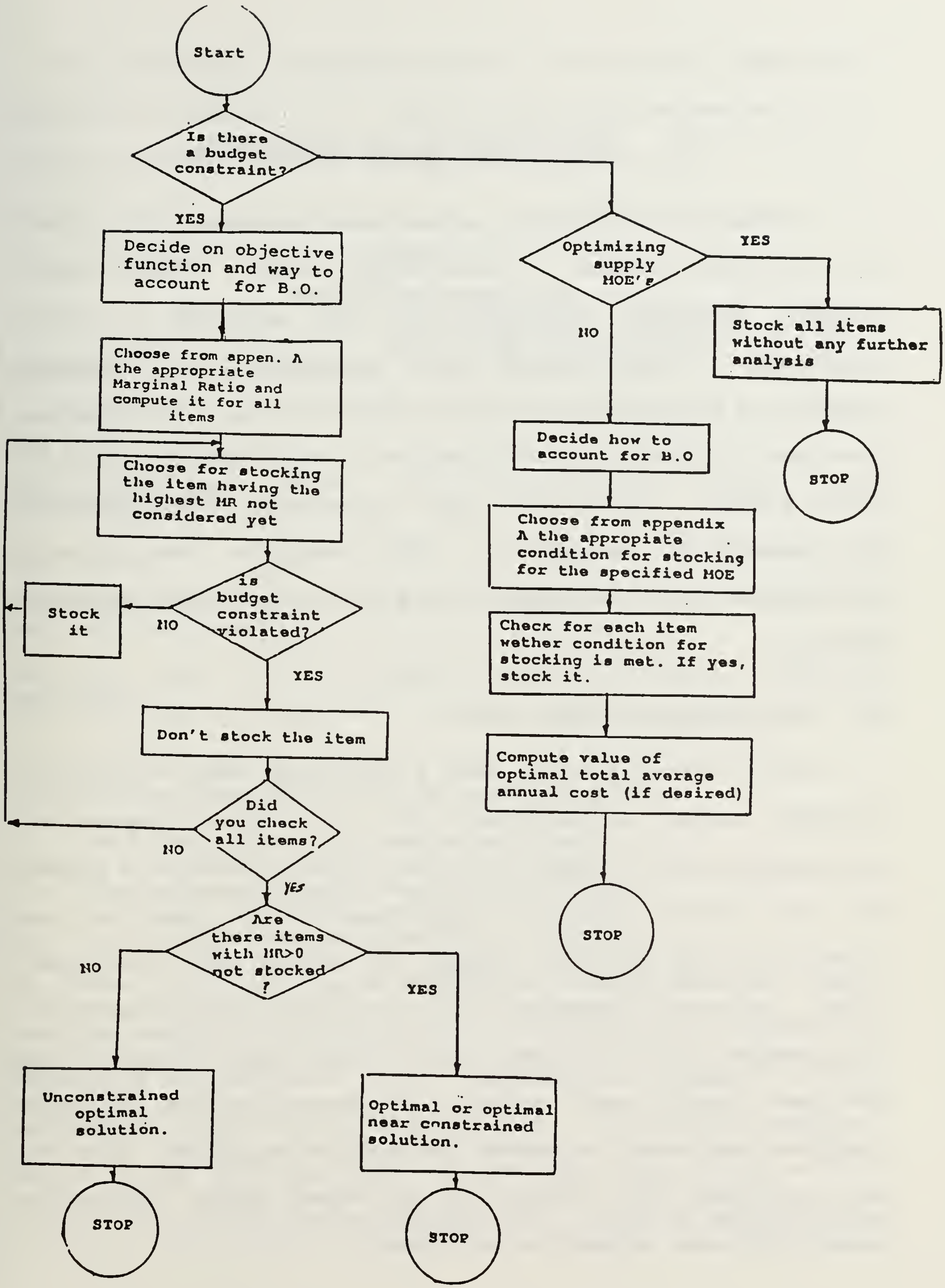

Figure 5.9 Solution Procedure for the Slow-Mover Stocking Decision.

VI. ILLUSTRATION OF THE MODEL

In Chapter V, different models were presented for solving the stocking problem of slow-moving items. These models support different objectives and can consider a budget constraint. This chapter uses a hypothetical numerical example to illustrate the way the models work. The example is designed to stress the differences in solutions provided by these models. It will also show one example of the benefits of sensitivity analysis. Such analysis can provide indications of how a solution is affected by model parameter changes.

A. THE EXAMPLE DATA SET

Let us assume that we have a situation where we need to consider annual replenishment for a kit of six expensive, slow-moving items. Assume also that after allocating a budget for the regular (medium to high demand rate) items, we are left with some residual budget for buying the "insurance" items. We need to remember that, according to our assumption, this budget is not a final one, but the first allocation as the new fiscal year begins. If demand during the year requires new funds, we assume they will be supplied. Thus, we are concerned with optimizing our first dollar allocation under different objective functions.

The relevant parameters values for the six items are provided in Table 6.1. The holding cost parameter h is assumed to have a value of 0.23.

TABLE 6.1

EXAMPLE PARAMETER SET

Parameter	Notation	Item A	Item B	Item C	Item D	Item E	Item F
Forecasted demand per year	D	1	1	0.5	1.5	0.2	0.5
Procurement lead time (years)	PCLT	2	2	1.5	2	2.5	0.2
Unit price	C	\$8K	25K	2K	10K	15K	10K
Backorder cost per unit per year	A′	\$4K	0.5K	2K	2K	10K	8K
Backorder cost per unit	A	\$2K	0.2K	3K	4K	4K	3K

Three budget condition will be considered:

- No budget limit (unconstrained case).
- Provisioning budget limited to \$15,000.
- Provisioning budget limited to \$25,000.

It turns out that the unconstrained annual expected costs problem can be solved by meeting the conditions derived in Section A of Chapter 5 or by computing the Marginal Ratios presented in Section B of Chapter 5. A positive Marginal Ratio value is equivalent to meeting the condition for stocking in the unlimited budget case. This is explained by

the fact that a positive Marginal Ratio means that $cost_i(1) < cost_i(0)$ (less costly to stock) which is exactly what the conditions for case C in Tables 5.1, 5.2 and 5.3 mean. We take advantage of this fact and just use the Marginal Ratio to determine the solution to the model for the three budget conditions given above.

B. COST OPTIMIZATION

1. EBO Model

Table 6.2 shows the analysis and solution guidelines for this case.

TABLE 6.2

ANALYSIS FOR COST MINIMIZATION WHEN EBO IS USED

Item	Demand During PCLT	Unit Price ($)	cost(0) ($)	cost(1) ($)	MCR	Ranking for Stocking
A	2	8,000	2,000	1,978	0.003	3
B	2	25,000	200	951	-0.03	-
C	0.75	2,000	1,500	1,009	0.245	1
D	3	10,000	6,000	5,816	0.018	2
E	0.5	15,000	800	2,407	-0.107	-
F	0.1	10,000	1500	2,223	-0.072	-

The unconstrained solution is to stock A, C and D which yields an optimal annual average cost of $11,303. This

solution costs $20,000 to procure and is also optimal even when the budget limitation is set to $25,000. If only $15,000 budget is available, the solution is to stock items C and D and the average annual cost will increase to $11,325.

2. TWUS Model

Table 6.3 represents the analysis and solution guidelines for this case.

TABLE 6.3

ANALYSIS FOR COST MINIMIZATION WHEN TWUS IS USED

Item	Demand During PCLT	Unit Price ($)	cost(0) ($)	cost(1) ($)	MCR	Ranking for Stocking
A	2	8,000	8,000	4,790	0.40	2
B	2	25,000	1,000	1,346	-0.01	-
C	0.75	2,000	1,500	662	0.42	1
D	3	10,000	6,000	4,214	0.18	3
E	0.5	15,000	5,000	3,158	0.13	4
F	0.1	10,000	800	2,120	-0.13	-

The unconstrained solution stocks items A, C, D, E and incurs an optimal average annual costs of $14,600 and total procurement costs of $35,000. If the budget available is only $25,000, the solution is to stock only A, C and D. The optimal average annual costs in this case will increase to $16,500. If a budget of only $15,000 is available, the solution will be to stock only items A and C. The average annual costs will

then increase to $18,200.

3. TWUS and EBO Combined Model

Table 6.4 shows the analysis and solution guidelines for this case.

TABLE 6.4

ANALYSIS FOR COST MINIMIZATION WHEN BOTH TWUS AND EBO ARE USED

Item	Demand During PCLT	Unit Price ($)	cost(0) ($)	cost(1) ($)	MCR	Ranking for Stocking
A	2	8,000	10,000	6,520	0.43	2
B	2	25,000	1,200	1,518	-0.012	-
C	0.75	2,000	3,000	1,453	0.77	1
D	3	10,000	12,000	9,915	0.21	3
E	0.5	15,000	5,800	3,473	0.15	4
F	0.1	10,000	2,300	2,263	0.003	5

The unconstrained optimal solution is to stock items A, C, D and E which costs $45,000. This result gives a minimum average annual cost of $24,800. When we limit the budget to $25,000, the average annual costs increase to $27,200 since we will stock only items A, C and D. When the budget is reduced to $15,000, we stock only items A and C and the average annual costs will increase to $29,300.

C. SUPPLY MOES

1. SMA Model

Table 6.5 shows the analysis and solution guidelines for this case. It should be remembered that maximizing SMA is equivalent to minimizing the expected number of backorders per year.

TABLE 6.5

ANALYSIS FOR MAXIMIZING SMA (MINIMIZING EBO)

Item	Demand During PCLT	Unit Cost ($)	EBO(0) units	EBO(1) units	MPR*	Ranking for Stocking
A	2	8,000	1	0.86	0.017	3
B	2	25,000	1	0.86	0.005	6
C	0.75	2,000	0.5	0.26	0.12	1
D	3	10,000	1.5	1.42	0.008	4
E	0.5	15,000	0.2	0.08	0.008	5
F	0.1	10,000	0.5	0.04	0.045	2

*The MPR is computed with unit costs divided by one thousand.

The unrestricted optimal solution is to stock all items and achieves an SMA of 25% (a budget of $70,000 is needed). When just $25,000 is available, the solution is to buy items A, C and F. In this case, SMA drops to 18%. When the initial annual provisioning budget is set to $15,000, the

solution is to buy items C and F. In this case, SMA drops further to 15%.

2. <u>MSRT Model</u>

Table 6.6 shows the analysis and solution guideline for this case.

TABLE 6.6

ANALYSIS FOR MSRT MINIMIZATION

Item	Demand During PCLT	Unit Cost ($)	MSRT(0) years	MSRT(1) years	MPR*	Ranking for Stocking
A	2	8,000	2	1.14	0.108	3
B	2	25,000	2	1.14	0.03	5
C	0.75	2,000	1.5	0.44	0.53	1
D	3	10,000	2	1.37	0.06	4
E	0.5	15,000	2.5	0.53	0.13	2
F	0.1	10,000	0.2	0.01	0.02	6

*The MPR is computed with unit costs divided by one thousand.

As expected, in the unconstrained problem we will stock all items and will used an initial budget of $70,000. The aggregate MSRT which results is 0.99 years (computed from Equation 4.6). When we limit the initial budget to $25,000, we will stock only items A, C, and E. This solution will achieve an aggregate MSRT of 1.39 years. With a budget of

only $15,000, the solution direct us to stock items A and C and the aggregate MSRT will increase to 1.48 years.

D. SUMMARY OF EXAMPLE OPTIMAL SOLUTION

Table 6.7 summarizes the optimal solutions for the examples under the various models and budget constraints presented in Section A through C.

TABLE 6.7

SUMMARY OF OPTIMAL SOLUTIONS FOR ALL CASES

Model Type			No budget constraint: Items and Perf.	No budget constraint: Initial Procurement Budget	$25,000 Budget Limit	$15,000 Budget Limit
COST MINIMIZATION	EBO	items perf.	A,C,D $11,300	$20,000	A,C,D $11,300	C,D $11,325
	TWUS	items perf.	A,C,D,E $14,600	$35,000	A,C,D $16,500	A,C $18,200
	TWUS +EBO	items perf.	A,C,D,E $24,800	$45,000	A,C,D $27,200	A,C $29,300
SUPPLY MAX OR MIN	Agg. SMA	items perf.	A,B,C,D,E,F 25%	$70,000	A,C,F 18%	C,F 15%
	Agg. MSRT	items perf.	A,B,C,D,E,F 0.99 years	$70,000	A,C,E 1.39	A,C 1.48

E. PARAMETRIC ANALYSIS OF BUDGET CONSTRAINT

The final step will be to conduct a parametric analysis to study how varying the level of the initial provisioning budget affects the optimal values of the various objective functions. This analysis provides valuable insights for the decision maker. For example, such an analysis can show us where we

could increase the initial provisioning budget by only a small amount and improve our objective function substantially.

1. Cost Models

Figures 6.1 and 6.2 show the optimal expected annual costs as a function of different initial annual provisioning budgets for the first two backorder cost alternatives.

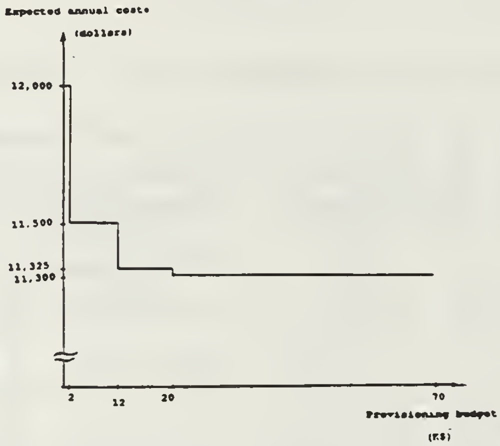

Figure 6.1. Optimal Solution to Annual Expected Costs When EBO is Used.

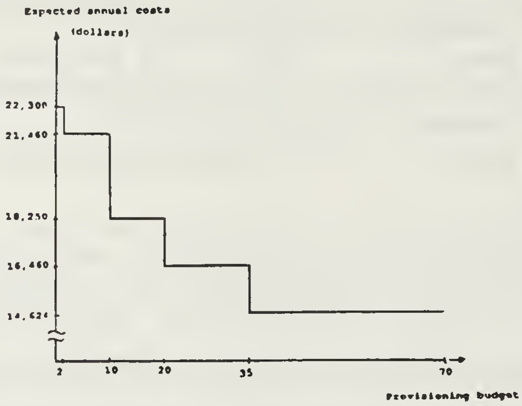

Figure 6.2. Optimal Solution to Annual Expected Costs When TWUS is Used.

In the first case, when EBO costs are used, just items A, C and D will be stocked in the unconstrained case (Table 6.7). This requires only $20,000. Beyond that point, stocking other items is not economical. In the second case, the unconstrained minimum annual costs are reached when items A, C, D and E are stocked at a cost of $35,000 (Table 6.7). Beyond that, no additional items should be stocked because they would cost more to stock than to backorder them.

2. <u>Supply Models</u>

a. SMA Model

Figures 6.3 and 6.4 show what happens to the aggregate SMA when it is the objective function and to the MSRT at different levels of an initial provisioning budget. The MSRT is shown only as additional information.

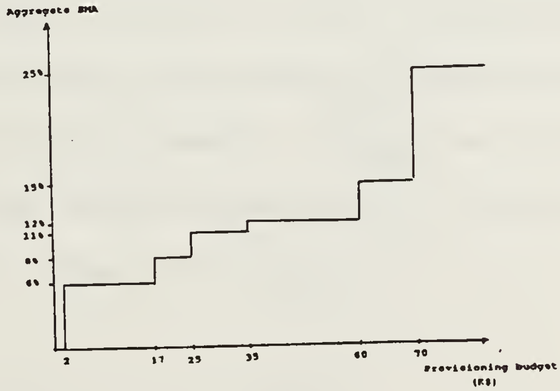

Figure 6.3. Aggregate SMA as a Function of the Budget Constraint for the SMA Model.

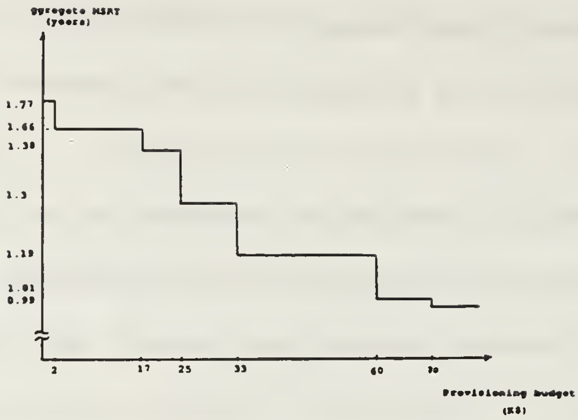

Figure 6.4 Aggregate MSRT as a Function of the Budget Constraint for the SMA Model.

b. MSRT Model

Figures 6.5 and 6.6 show what happens to the aggregate average MSRT objective function and SMA at different levels of the initial provisioning budget. The SMA is shown just as additional information.

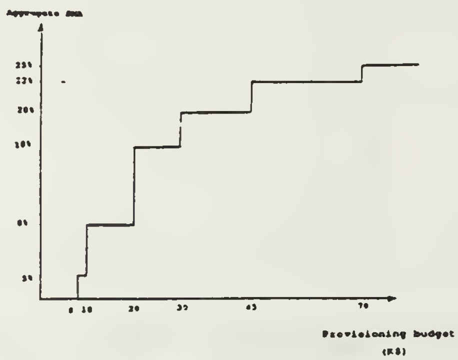

Figure 6.5 Aggregate MSRT as a Function of the Budget Constraint for the MSRT Model.

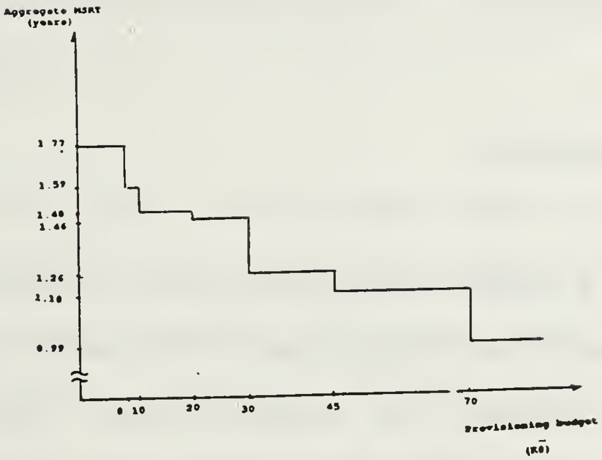

Figure 6.6 Aggregate SMA as a Function of the
Budget Constraint for the MSRT Model.

Figures 6.3 to 6.6 show us that, in both models, both the SMA and MSRT improve when the budget constraint is relaxed. There are still differences between the two models, especially in the high range of the provisioning budget constraint. In the example considered, these are considered to be minor. This may not, however, always be true since the two models can rank items differently in more complex situations where more items are involved.

VII SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

A. SUMMARY

This thesis develops some cost/performance continuous review models for high cost, low demand insurance items. The nature of these low demand items suggested two possible stocking alternatives. One was to stock just one unit and the other was not to stock the item at all. The reorder point in both cases was zero. The models optimized different objective functions under the assumption of steady-state conditions with annual demand being Poisson distributed. Both unconstrained and constrained optimization were considered. The constraint was the initial annual replenishment budget. The flow diagram shown in figure 5.9 and the different ratios and conditions presented in Appendix A, give a comprehensive procedure for how to determine the optimal stocking solution. An example was solved in chapter VI to demonstrate the solution process. Different parametric analyses can increase the understanding of the behavior of the optimal solution as a function of the parameter's value. One example of such an analysis is presented in chapter VI. It examined the effect of the initial annual budget constraint on the constrained problem of this thesis.

B. CONCLUSIONS

The analyses showed that in the cases where supply MOEs are considered, the objective function will always be improved by stocking as many items (one unit from each item) as we can ,depending on the budget constraint. When expected annual costs are used as MOE, this might not always be the case, because stocking one unit of an item is not necessarily more economical than not having it on the shelf.

The cost models are heavily dependent on the numeric values of the shortage cost parameters associated with the items. This can be viewed as a drawback of these models since these parameters may be difficult to estimate. The supply performance models are not influenced by these parameters. They only depend on parameters such as annual demand, procurement lead-time and the unit cost of the item. These are typically available from historic data and are therefore more easily estimated with much more confidence.

The supply MOEs enjoy an advantage over the cost models. In the military environment they are often preferred because they are associated with trying to keep the performance of weapon systems as high as possible. In many cases we are willing to accept a more expensive solution if it brings the system's

performance measure up. In other words, we are willing to hold the unit, even if it is expected to cause higher expense than not stocking it, to achieve better supply MOEs and operational MOE's.

C. RECOMMENDATIONS

This thesis effort was the beginning of the development of models for managing slow moving items. Therefore it is too early to recommend one for use by the Israeli Navy (IN). In particular all objective functions presented in this thesis will be new to that Navy and will need to be examined carefully before further model development steps are taken. As a consequence, three steps are recommended at this stage. They are :

1. Introduce the models to the I.N. and conduct a preliminary analysis to check the appropriateness of MOEs such as MSRT and SMA and see if they will fit needs and comply with the general philosophy of managing its inventory.

2. For the cost models, it may be feasible to develop those difficult costs parameters such as backorder costs for items/technological groups (or other grouping methods)

as is done now by the U.S Navy. In addition, a study will need to be conducted to determine a method for assessing the holding costs in the I.N.

3. Perhaps the logic used in this thesis could be used for other populations of items. Maybe the models can work also on cheap, high demand consumables in addition to slow-moving insurance type items . The basic steady-state formulas for any reorder point and order quantity have been derived in chapter IV. These could be used with more elaborate marginal analysis such as that described in reference 10, for general stocking of consumables.

Hopefully after these three steps are done and enough evidence has been obtained on how successful the models might be, we may be able to begin implementation of one or more of the models derived in this thesis.

APPENDIX A

TABLE A.1

SUMMARY OF STOCKING CONDITIONS AND MARGINAL RATIOS

Type of Problem	Objective Function Type	Form of shortage cost of supply performance	Condition/Marginal ratio's calculation for stocking a Slow Mover
UNCONSTRAINED PROBLEM	Cost Minimization	EBO	$D > \frac{C \cdot h}{A}$
		TWUS	$D.PCLT > -\ln \left[\frac{A'}{C \cdot h + A'}\right]$
		EBO + TWUS	$D > \frac{p(0)\,[C \cdot h + A'] - A'}{A \cdot p(0)}$
CONSTRAINED PROBLEM	Cost Minimization	EBO	$MCR = \frac{e^{-D \cdot PCLT}(A \cdot D - h \cdot C)}{C}$
		TWUS	$MCR = \frac{A' - e^{-D \cdot PCLT}[h \cdot C + A']}{C}$
		EBO +TWUS	$MCR = \frac{e^{-D \cdot PCLT}(A \cdot D - A' - h \cdot C) + A'}{C}$
	Supply MOEs	SMA	$MPR = \frac{D \cdot e^{-D \cdot PCLT}}{C}$
		MSRT	$MPR = \frac{1 - e^{-D \cdot PCLT}}{C}$

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